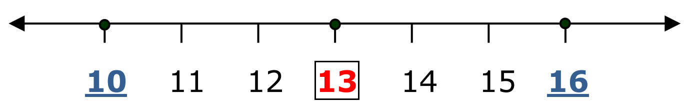
CH NN – MIDPOINT ON THE LINE AND IN THE PLANE

❑ MIDPOINT ON THE LINE

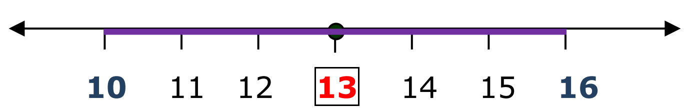
Here's a question for you: What number is *midway* between 10 and 16? You probably know that the number is 13. Why? Because 13 is 3 units away from 10, and 13 is also 3 units away from 16.



Now we need a simple way to find the number that is midway between any two numbers, even when the numbers are not nice, or worse yet, when there are variables involved. Notice this: If we take the **average** (officially called the *arithmetic mean*) of 10 and 16 — by adding the two numbers and dividing by 2 — we get

$$\frac{10+16}{2} = \frac{26}{2} = \mathbf{13}\text{, the midway number}$$

Let's rephrase what we've done with some new terminology. Consider the *line segment* connecting 10 and 16 on the number line:



We can now refer to the 13 as the ***midpoint*** of the line segment connecting 10 and 16.

What is the *midpoint* of the line segment connecting –2.8 and 14.6? Just calculate the average of –2.8 and 14.6:

$$\frac{-2.8+14.6}{2} = \frac{11.8}{2} = \mathbf{5.9}$$

> When you see the term ***midpoint***, think *average*!

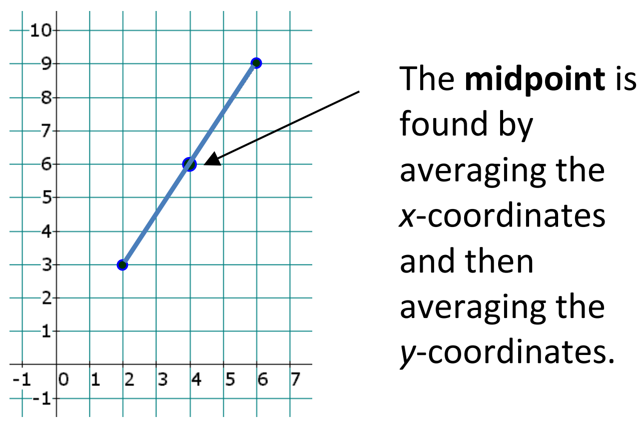
Homework

1. Find the **midpoint** of the line segment connecting the two given numbers on a number line:

a. 10 and 20	b. 13 and 22	c. –8 and –26
d. –3 and 7	e. –7 and 6	f. π and –π
g. –21 and –99	h. 0 and 43	i. –50 and 0
j. –44 and 19	k. –41 and 88	l. 3*x* and –*x*

❑ MIDPOINT IN THE PLANE

Now for the more important question: Consider the two points (2, 3) and (6, 9) in the plane and the line segment that connects them. We need to figure out what point is the *midpoint* of the line segment connecting the two points. Recall the advice given above: When you see midpoint, think *average*. So the *x*-coordinate of the midpoint is the average of the *x*-coordinates of the two endpoints:



$$x = \frac{2+6}{2} = \frac{8}{2} = \mathbf{4}$$

And the *y*-coordinate of the midpoint is the average of the *y*-coordinates of the two endpoints:

$$y = \frac{3+9}{2} = \frac{12}{2} = \mathbf{6}$$

We conclude that the midpoint is **(4, 6)**. That's all there is to it. Now let's do a complete example without plotting any points or drawing any segments.

<u>EXAMPLE 1:</u> **Find the midpoint of the line segment connecting the points (–42, –33) and (90, –10).**

<u>Solution:</u> No graphing needed — we have a formula. The x-coordinate of the midpoint is found by averaging the x-coordinates of the two given points:

$$x = \frac{-42+90}{2} = \frac{48}{2} = 24$$

The y-coordinate of the midpoint is found by averaging the y-coordinates of the two given points:

$$y = \frac{-33+(-10)}{2} = \frac{-43}{2} = -\frac{43}{2}$$

The midpoint is therefore the point

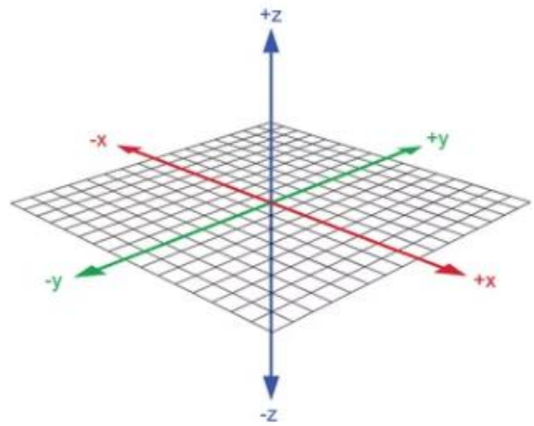
$$\boxed{\left(24, -\frac{43}{2}\right)}$$ or, (24, –21.5)

Homework

2. Find the **midpoint** of the line segment connecting the given pair of points:

a. (–2, 5) and (2, 7)
b. (0, 1) and (0, 6)
c. (–5, 8) and (–5, –8)
d. (–2, 7) and (5, –3)
e. (–9, 2) and (–13, –40)
f. (0, 0) and (–6, –9)
g. (5, 4) and (5, 4)
h. (14, 0) and (0, –9)
i. (8, 8) and (–19, –19)
j. (π, 0) and ($-\pi$, 0)
k. $(0, \sqrt{2})$ and $(0, -\sqrt{2})$
l. (a, b) and (c, d)
m. (a, b) and (a, $-b$)
n. ($3a$, $3b$) and ($-3a$, b)

❑ *TO ∞ AND BEYOND*



A point on a 1-dimensional line can be described by a single number, for example, 7.

A point in a 2-dimensional plane can be described by an ordered pair, for example, (2, –9).

A point in 3-dimensional space (which has an x-axis, a y-axis, and a ***z*-axis**), can be described by an ordered triple, for example (1, –5, 12).

Find the **midpoint** of the line segment connecting the points (2, –3, 7) and (–5, 17, 20).

Solutions

1.

a.	15	b.	17.5	c.	–17	d.	2	e.	–0.5	f.	0
g.	–60	h.	21.5	i.	–25	j.	–25	k.	23.5	l.	x

2.

a.	(0, 6)	b.	(0, 7/2)	c.	(–5, 0)
d.	(3/2, 2)	e.	(–11, –19)	f.	(–3, –9/2)
g.	(5, 4)	h.	(7, –9/2)	i.	(–11/2, –11/2)
j.	(0, 0)	k.	(0, 0)	l.	$\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$
m.	$(a, 0)$	n.	$(0, 2b)$		

"What sculpture is to a block of marble, education is to the human soul."

Joseph Addison